



RESEARCH AND DEVELOPMENT PROCESS OF PENETRATE CORE LITERACY-THE ROTATION AND TRANSFORMATION OF GRAPHICS AS AN EXAMPLE

Yuchan Su¹, Haiyan Zhang²

^{1,2}Department of Mathematics and Statistics, Guangxi Normal University, Guilin, and China, 1106481531@qq.com

Abstract

This paper focuses on solving problems from line and difference problems, proving angle problems and common vertex problems using rotational transformation ideas to expand the problem-solving process and develop students' core literacy. After analyzing the mathematics test questions, it is not difficult to find that the rotation of a figure is often tested in a comprehensive expansion. The test questions focused on the problems related to angles or line segments when the figure is rotated to a particular position, integrating the dynamic geometry of change and invariance, geometric conjecture, drawing, reasoning, proof and calculation ability, as well as innovation and practical ability of students' experience of the operation activities related to the transformation of a figure. The solution of such problems requires intuitive imagination, logical reasoning and mathematical literacy. After rotation, the figure and the original figure are congruent, therefore, the method of rotational change becomes a new way of thinking in solving mixed problems with complex figures, and it is easier for students to understand the idea of doing the problem by rotating and transforming the figure to construct auxiliary lines.

Keywords: Rotation transformation; core literacy in mathematics; junior high school mathematics

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INTRODUCTION

The Compulsory Education Curriculum Standards for in the mathematics curriculum clearly require the purpose of the exercise of core literacy for students (Ministry of Education of the People's Republic of China, 2022; Tang et al., 2023). In fact, it is to reserve the ability for students to adapt to the needs of social development, but also will enable students to comprehensively improve the comprehensive quality in mathematics learning. Core literacy will allow students to master

mathematical knowledge at the same time, can be led by a stream of ideas, which promotes the overall development of students in the mind (Ministry of Education of the People's Republic of China, 2011). Teachers not only focus on students' learning in the process of teaching, but also on the cultivation of mathematical ideas (Li et al., 2019; Pereira, 2023). Our secondary school mathematics competitions also include elementary geometry proofs as part of the competition, and the grand finale of the Chinese examination is the one that turns the idea of geometric transformations from a simple and easy to understand operation into a variable and difficult problem that everyone wants to avoid. However, the idea of graphical transformation is seldom incorporated into geometric problem solving, as well as the inability to fully utilize the nature of rotational variation in the rotational transformation problem type. Rotational transformation of graphs is the best thinking gymnastics for spatial thinking ability training, and its position in junior high school mathematics teaching is very important. Teachers should guide students to explore the characteristics of rotation and transformation of graphs from different perspectives, systematize and rationalize students' scattered and perceptual knowledge, realize the integration of knowledge and develop spatial concepts from life examples (Chen, 2019). In the teaching process, establish the number sense to prove the quantitative relationship and the solution of corresponding sides and angles, use the spatial concept and reasoning ability to explore the ideas by constructing auxiliary lines and sympathetic reasoning through rotational transformation. Consciously use the concept of rotation, principles and methods to solve elementary geometric transformation problems, categorize the topics and discover the laws of change, so as to cultivate students' sense of application and innovation, and exercise the four. Base 4 skills to increase students' interest and confidence in mathematics (Zhiyan, 2019).

Theoretical Basis Of The Study

Rotation Transformation Rotational Transformation Definition

The rotation transformations are explained in the compulsory education textbook of mathematics, grade 7, published by Hunan Education Press (Shijian & Chufang, 2012). The definition of rotational transformation is: each point on a plane figure F is rotated by an angle around a certain point in this plane to obtain the figure F' , and this transformation of the figure is called rotation (Shijian & Chufang, 2012). As in Figure 1

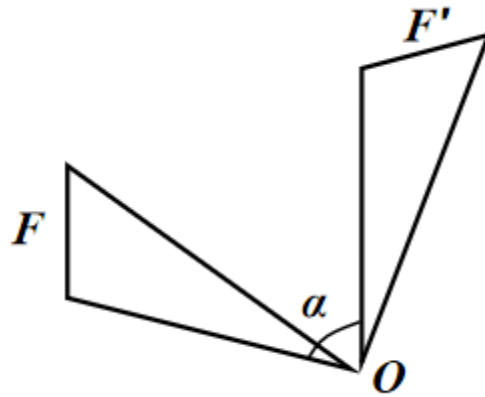


Figure 1

Rotational Transformation Properties

By definition the following properties of the rotational transformation are available.

Property 1 The distances from the corresponding points to the center of rotation are equal.

Property 2 The figures before and after rotation are identical.

Property 3 The angles formed by the line between two sets of corresponding points and the center of rotation are equal.

Nature 4 The center of rotation is the only point that does not move.

Rotation Transformation

In the 2011 edition of the new curriculum standards for middle school mathematics, it is explained that in the mathematics curriculum, emphasis should be placed on developing students' number sense, symbolic awareness, spatial concepts, geometric intuition, data analysis concepts, arithmetic skills, reasoning skills, modeling ideas, application awareness, and innovation. Graphical rotation and transformation questions are an important part of the knowledge of graphs and geometry after the new curriculum change. Teachers should also enhance students' understanding of graphical rotation transformation problems while downplaying the proof process (Wu, 2021).

Research Question

The Compulsory Education Mathematics Curriculum Standards (2022 Edition) stipulates the content of graphical transformations such as graphical rotations. Although the phenomenon of rotation can be seen everywhere in life, how to use graphical rotation reasonably and effectively to solve problems is something we need to think about and explore. To solve mathematical problems requires mathematical knowledge, but also requires mathematical thinking. Research shows that students usually use the method of congruent transformation to solve the graphical rotation and

transformation problems, but there are two difficulties: first, it is difficult to find the basic model of the topic; second, it is not easy to grasp the direction of the solution without a deep understanding of the theory of congruence of associated triangles (Biyu & Liya, 2019). In the following, we will talk about how to use graphical rotation and congruent transformation to solve problems effectively based on how to understand the definition of graphical rotation and its properties, expand the problem solving ideas, experience the changes before and after the graphical movement and changes, feel the charm of mathematics, gradually form the concept of space and geometric intuition, and cultivate students' core mathematical literacy.

Research Objective

Inspire teachers how to penetrate the core mathematical literacy in the teaching of rotational transformation problems, and students, on the basis of learning the definition and properties of rotational transformation, teach students to use rotational transformation to solve line and difference problems, angle equality problems, solve common vertex problems, learn to change one problem and solve multiple problems, in these topics, starting from the topic problem, activate the condition information, through the known condition of the sides are equal, and then find that there are special angles or have the same corresponding angles equal, and finally determine a target triangle to rotate the triangle by a certain angle with a fixed point. It is possible to reason about the practice of auxiliary lines, to make the addition of auxiliary lines traceable, to break through the construction difficulties, and to improve students' geometric intuition and reasoning skills.

RESEARCH METHODOLOGY

First of all, this study was conducted to understand the current situation of the development of core literacy of students in the teaching of graphical rotational transformation solution and geometry problem solving at home and abroad by reviewing relevant books, journals, dissertations, and internet resources, and to reflect on it in depth, so as to provide effective guidance for teachers on how to penetrate the core literacy of mathematics in the teaching of rotational transformation problems.

The case study method is combined with the definition and properties of graphical rotation and transformation to the analysis of the Chinese mathematics examination questions, examining the graph rotated to a particular position related to the angle or line segment, focusing on the students' experience of manipulative activities related to graphical transformation, thus solving such problems, for intuitive imagination, logical reasoning and other mathematical literacy requirements

are extremely high. The method of change by means of rotation is a new way of thinking in solving problems with a mixture of complex figures, and it is easier for students to understand the idea of doing problems by rotating and transforming figures to construct auxiliary lines.

RESULT AND DISCUSSION

Application of rotational transformations to plane geometry problems

Solving line sum and difference problems using rotational transformations

Example 1: (2020 - Banan District Independent Admissions) It is known that in rectangle ABCD, $AB = 2$, point E is on side BC and $AE \perp DE$, $AE = DE$, point F is a point on the extension line of BC, AF and DE intersect at point G, $DH \perp AF$, the vertical foot is H, the extension line of DH and BC intersect at point K.

(1) (a) Find the length of AD as in Figure 2.

As shown in Figure 3, connect KG, and prove that $AG = DK + KG$.

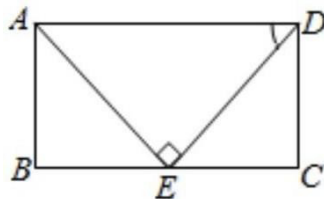


Figure 2

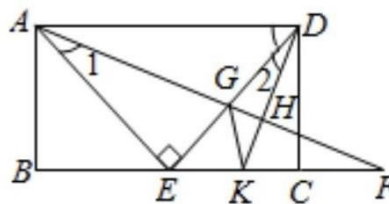


Figure 3

Topic Analysis: The types of questions are all proofs of the addition and subtraction transformations of line segments. A common method to solve such problems is to make appropriate auxiliary lines by rotational transformation and solve them with the help of congruent or similar triangles (Qiang-Hua, 2022). One can think of one of the line segment substitutions into another line segment, it makes the line segment ① = line segment ②, then such ways are: 1. two figures are congruent; 2. the basic knowledge of the basic figure (such as isosceles triangle two sides are equal or the sides of the square are equal), through such line segment transformation to contribute to the proof of the topic of line segment addition and subtraction. Generally speaking, the line segment sought by the topic in a different line segment, or different graphics, the line segment transformation to the same line segment or graphics, and so how to do it? The use of rotational transformation as an entry point, and then use its properties to solve the problem. It is also an examination of the basic knowledge and skills of shapes such as squares or rectangles and triangles,

testing students' number sense, reasoning ability and spatial concepts of elementary geometric transformations.

1. Solution:

$\because$ quadrilateral ABCD is a rectangle,

$\therefore AB=CD, \angle B=\angle C=90^\circ,$

$\because AE=ED,$

$\therefore \text{Rt}\triangle ABE \cong \text{Rt}\triangle DCE (\text{HL}),$

$\therefore BE=CE, \angle AEB=\angle DEC,$

$\because AE \perp DE,$

$\therefore \angle AED=90^\circ,$

$\therefore \angle AEB=\angle DEC=45^\circ,$

$\therefore \angle BAE=\angle AEB=45^\circ,$

$\therefore BE=AB=2,$

$\therefore AD=BC=2BE=4,$

(2) Method 1.

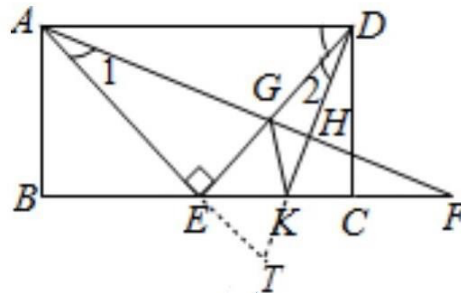


Figure 4

Analysis of conventional method: As in Figure 4, extend AE to intersect the extension of DK in T. Use the nature of congruent triangle to prove $\triangle AEG \cong \triangle DET$ (ASA), get $AG=DT$; then use $\triangle KEG$ and $\triangle KET$ congruent, get $GK=KT$ that can solve the problem.

Solution:

As in Figure 4, extend AE to intersect the extension of DK in T

$\because DH \perp AF,$

$\therefore \angle DHG=\angle AEG=90^\circ,$

$\because \angle AGE=\angle DGH,$

$\therefore \angle 1=\angle 2,$

$\because \angle AEG = \angle DET = 90^\circ, AE = DE,$
 $\therefore \triangle AEG \cong \triangle DET \text{ (ASA)},$
 $\therefore EG = ET, AG = DT,$
 $\because \angle KEG = \angle KET = 45^\circ, EK = EK,$
 $\therefore \triangle KEG \cong \triangle KET \text{ (SAS)},$
 $\therefore GK = KT,$
 $\therefore DT = DK + KT = DK + GK,$
 $\therefore AG = GK + DK,$

Method 2.

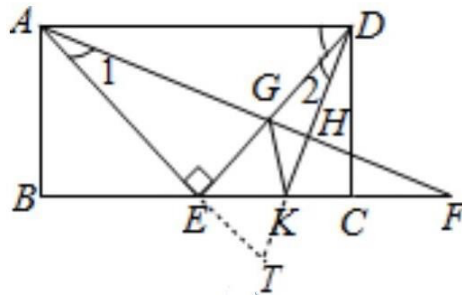


Figure 5

Analysis using the method of rotation and transformation: The second sub question of this problem requires proof that $AG = DK + KG$. As in Figure 5. First, students substitute a line segment in AG , DK , KG and then establish the relationship, from which they can think of congruent figures with equal corresponding sides, thus solving the problem by the nature of the rotation and transformation of the figure. Second, by the title, given and geometric proof can be found: $\angle 1 = \angle 2$, $AE = ED$, then change one of the line segments switch over to the same line segment. If AG and DK are co-linear, the sum-and-difference relationship may hold. Then venture to guess where $\triangle AED$ will be after rotation; rotate $\triangle AED$ 90° clockwise around the point E , get the point T and draw the auxiliary line, we can get AG transformed to DT and common line with DK ; finally, the geometric transformation gets that $\triangle KEG \cong \triangle KET$, so that KG is related to AG , DK . Then prove that $GK = KT$ by $\triangle KEG \cong \triangle KET$, and thus prove that $AG = DK + KG$.

Solution:

$\because DH \perp AF,$
 $\therefore \angle DHG = \angle AEG = 90^\circ,$
 $\because \angle AGE = \angle DGH,$

$$\therefore \angle 1 = \angle 2,$$

Rotate $\triangle AEG$ clockwise by 90° about E to get $\triangle ETD$, then $\triangle AEG \cong \triangle ETD$

$$\therefore EG = ET, AG = DT,$$

$$\because \angle KEG = \angle KET = 45^\circ, EK = EK,$$

$$\therefore \triangle KEG \cong \triangle KET \text{ (SAS)}$$

$$\therefore GK = KT,$$

$$\because DT = DK + KT = DK + GK$$

$$\therefore AG = GK + DK.$$

Example 2: (2020-Anhui) Figure 6 shows that quadrilateral ABCD is a rectangle with point E on the extension of BA and AE = AD. EC and BD at point G, and AD at point F, AF = AB.

(1) If AB = 1, find the length of AE.

(2) As in Figure 7, connect AG and show that $EG - DG = \sqrt{2}AG$.

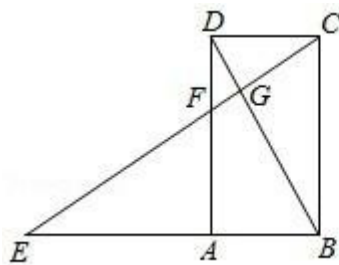


Figure 6

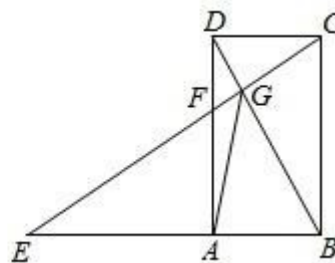


Figure 7

(1) Solution:

$\because$ quadrilateral ABCD is a rectangle,

$$\therefore AE \parallel CD,$$

$$\therefore \angle AEF = \angle DCF, \angle EAF = \angle CDF,$$

$$\therefore \triangle AEF \sim \triangle DCF,$$

$$\therefore \frac{AE}{DC} = \frac{AF}{DF},$$

$$\therefore AE \cdot DF = AF \cdot DC,$$

Suppose $AE = AD = a$ ($a > 0$),

$$\therefore a \cdot (a - 1) = 1,$$

$$\therefore a^2 - a - 1 = 0,$$

$$\therefore a = \frac{1+\sqrt{5}}{2} \text{ or } \frac{1-\sqrt{5}}{2} \text{ (give up),}$$

$$\therefore AE = \frac{1+\sqrt{5}}{2}$$

(2) As shown in Figure 8.

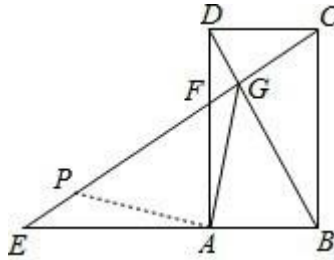


Figure 8

Analysis using the rotational transformation method: The second sub-question of this question asks to prove that $EG - DG = \sqrt{2} AG$. You can use the same idea as the previous problem to solve the problem by rotating and transforming. First of all, find a way to establish the relationship between EG, DG, and AG by rotating them, and make a

conjecture from "EG-DG": if EG and DG are co-linear, the sum and difference relationship is possible. Secondly, after rotating $\triangle ADG$ counterclockwise by 90° around point A, we get EG, DG co-linear; finally, after rotation, using the properties of triangles, we conclude that $\triangle PAG$ is an isosceles right triangle, which leads to the quantity relationship of $EG - DG = \sqrt{2} AG$.

Solution:

Rotate $\triangle ADG$ around point A counterclockwise by 90° to get $\triangle AEP$,

$$\therefore \triangle ADG \cong \triangle AEP$$

$$\therefore AP = AG \text{ and } \angle EAP = \angle DAG.$$

$$\therefore \angle PAG = \angle PAD + \angle DAG = \angle PAD + \angle EAP = \angle DAE = 90^\circ.$$

$\therefore \triangle PAG$ is an isosceles right triangle.

$$\therefore EG - DG = EG - EP = PG = \sqrt{2} AG.$$

In the problem of proving the equivalence of addition and subtraction of line segments, solving such problems stimulates the sense of inquiry, deep thinking, divergent thinking and bold conjecture. The practice of auxiliary lines from the perspective of rotation is well documented, and

the use of the nature of rotation helps to construct auxiliary lines, which is conducive to the cultivation of students' logical reasoning and spatial imagination.

Discovering potential relationships of quantities, constructing common lines of line segments or within the same figure, reasoning about addition and subtraction symbolic awareness to get general conclusions. Understanding the use of symbols to facilitate mathematical expression and thinking is one of the most important items in building symbolic awareness. Not only that, teachers should guide students in the classroom to learn to construct auxiliary lines to understand or express quantitative relationships and to promote the development of number sense. Use symbols to express the patterns and quantitative relationships of quantities as a way to draw conclusions (Zhang, 2023).

Solving angle equality problems using rotational transformations

Example 1: As in Figure 9, it is known that $\triangle ABC$, $\angle ACB = 90$ degrees, $CD \perp AB$, D is the vertical foot, extend CB to E , so that $EB = CB$, link AE intersects the extension of CD in F , link FB , if at this time $AC = EC$, Proof: $\angle ABC = \angle EBF$.

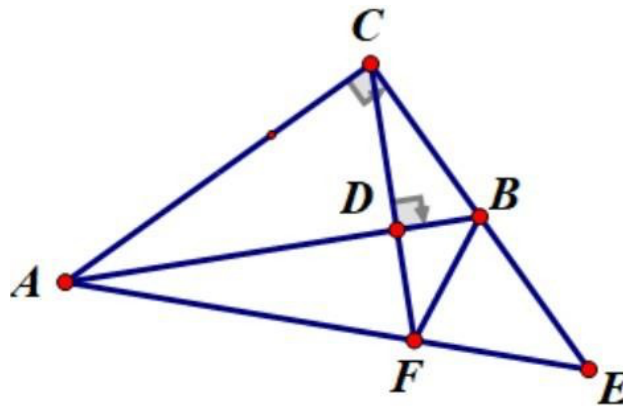


Figure 9

Topic Analysis: The problem is to prove that the angles of two angles are equal, and students can easily associate it with proving that two triangles are congruent so that the corresponding angles are equal (Jinlu, 2023). However, recall that the triangle in which the two angles are located is obviously not congruent.

At this point, you have to use the auxiliary line to construct, you can make the auxiliary line from the known conditions of equal line segments or equal angles to form the corresponding side or corresponding angle, using a triangle as the reference to construct and make the auxiliary line (Wang, 2023). There are many different ways to solve this problem, and there are many ways to use

the auxiliary line from different known conditions as the entry point, which examines the students' creativity and logical reasoning ability.

Method 1.

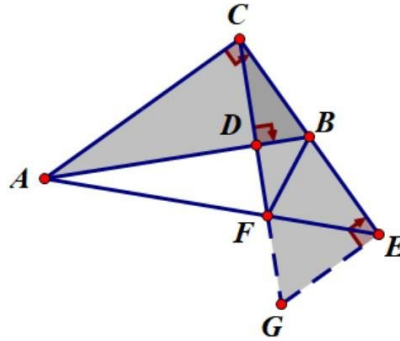


Figure 10

Conventional method analysis: As in Figure 10, using $CE = AC$ as the corresponding side, choose $\triangle ABC$ as the target triangle, because If $\angle BAC = \angle GCE$, then we have to construct a right triangle with CE as side and $\angle GCE$ as congruent to it. Therefore, the auxiliary line is to make $EG \perp CE$ through point E and cross the extension of CF at point G . Then $\triangle ABC \cong \triangle GEC$, the problem is transformed to prove that $\angle EBF = \angle G$. Secondly, prove that $\triangle EFB \cong \triangle EFG$. Thus, $\angle FBE = \angle G$.

Solution:

$\because$ the auxiliary line is to make $EG \perp CE$ through point E and cross the extension of CF at point G

$$\therefore \angle ACB = \angle CEG = 90^\circ,$$

$$\because CD \perp AB,$$

$$\therefore \angle CAB + \angle ACD = 90^\circ,$$

$$\because \angle ACB = 90^\circ,$$

$$\therefore \angle ACD + \angle GCE = 90^\circ,$$

$$\therefore \angle CAB = \angle GCE,$$

$$\because AC = CE,$$

$$\therefore \triangle ACB \cong \triangle CEG,$$

$$\therefore BC = GE, \angle ABC = \angle CGE,$$

$$\because AC = CE, \angle ACE = 90^\circ,$$

$$\therefore \text{Rt}\triangle ACE \text{ is an isosceles right triangle,}$$

$$\therefore \angle CAE = \angle CEA = 45^\circ,$$

$$\begin{aligned}
&\because \angle CEG = 90^\circ, \\
&\therefore \angle FEG = 45^\circ, \\
&\because BC = BE, \\
&\therefore BE = GE, \\
&\because FE = FE, \\
&\therefore \triangle FEG \cong \triangle FEB, \\
&\therefore \angle FBE = \angle FGE, \\
&\therefore \angle ABC = \angle EBF.
\end{aligned}$$

Method 2.

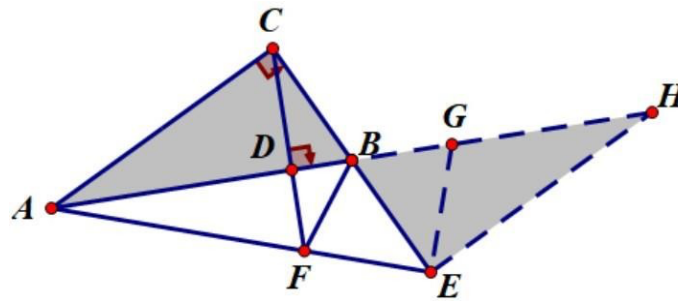


Figure 11

As in Figure 11, analysis using the rotation transformation method: the problem is to prove that two angles are equal, then the solution is through the triangles are congruent to prove that the corresponding angles are equal (Zhiyan, 2019). First, the line with equal $CB=BE$ is the entry point, and $\triangle ABC$ is chosen as the target triangle, if rotating 180° clockwise around point B, $\triangle BHE$ is obtained, then the problem requires $\angle ABC = \angle EBF$ to become $\angle ABC = \angle EBH$. Second, choose $\triangle EFC$ as the target triangle, and then construct a triangle with $\angle H$ and $\triangle EFC$ congruent. The result is that $\triangle CFE \cong \triangle EGH$ according to "SAS" and $\triangle FEB \cong \triangle BEG$ according to "SAS", so $\angle ABC = \angle EBF$.

Solution: Rotate $\triangle ABC$ clockwise 180° about B to get $\triangle BHE$, then $\triangle ABC \cong \triangle BHE$

$$\therefore AC = EH = CE, \angle CAB = \angle EHB = \angle FCE.$$

On the edge of BH, make a point G such that $GH = AF$ and connect GE

$$\therefore \triangle CFE \cong \triangle EGH \text{ (SAS), that is, } EF = GE, \angle AEC = \angle GEH,$$

$$\therefore \triangle FEB \cong \triangle BEG.$$

$$\therefore \angle ABC = \angle EBH, \text{ so } \angle ABC = \angle EBF.$$

The problem shows a multi-solution approach, and different known conditions are used as entry points to obtain different solutions, which is a way to develop students' divergent and creative

thinking (Li, 2023). The cultivation of students' sense of innovation becomes a basic task, and innovative thinking is especially important in the process of problem solving.

Students open their minds, put their knowledge of plane geometry into full understanding and reach a coherent approach to problem solving. Different angles and levels of analysis are helpful to train the ability to distribute thinking and association. Therefore, teachers should guide students to categorize the topics of rotation types, discover the laws of change, build and solve models, get the solutions of various types of problems of rotation models in different categories, find the results, and discuss the results to complete the meaning of multiple solutions and multiple solutions to one. For solving geometry problems, identifying or constructing basic shapes and using existing conclusions is the source of problem solving, as the saying goes, if you have a method in mind, your interest will naturally grow (Cheng, 2022).

Solving co-vertex problems using rotational transformations

Example 1: As shown in the figure 12, point P is a point inside square ABCD, and the distances from point P to points A, B and C are $2\sqrt{3}$ 、 $\sqrt{2}$ 、4, then the area of square ABCD is the $14+4\sqrt{3}$.

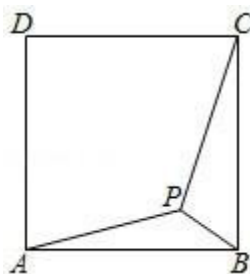


Figure 12

Topic Analysis: This question examines the common vertex rotation model, the question AP, CP, BP has a common point P, if the three line segments can form a triangle, the three line segments are concentrated together, then the topic is well solved (Miao, 2023). We can think of constructing the rotation of the common vertex. This question is a computationally intensive problem in elementary geometric transformation, so it tests the students' ability of arithmetic and data analysis; for $\triangle APB$ or $\triangle CPB$, the two triangles can be rotated with each other and then one of the sides can be put together, so it tests the students' ability of geometric intuition.

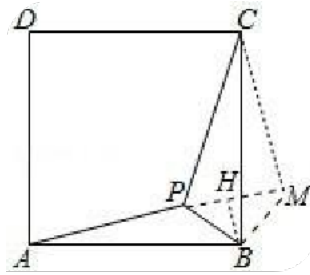


Figure 13

Analysis: As shown in the figure 13, $AB = BC$, rotate $\triangle ABP$ clockwise a round point B, then AP, CP and BP can be separated by rotation, only CP is not rotated out at this time. Reviewing the problem, we find that the information obtained after the rotation makes it impossible to find out the side lengths, which are obtained by the nature of the rotation transformation $\angle PBM = 90^\circ$ and $BP = BM$, which leads to an isosceles right triangle, then we can connect PM and find the length of PM. Immediately after that, we can see that all sides of $\triangle PCM$ are known, so we can conclude that this triangle is a right triangle with special angles by the inverse theorem of the Pythagorean theorem (Tian, 2023). Returning to in the problem itself, if the length of the side of AB is required, the lengths of the other two sides must be known. First of all, for the hypotenuse: A, P, M whether the common line becomes the first, this is the problem of calculating each angle, prove that $\angle APM$ is a flat angle. Second, after finding out the angles, there is still no way to find out the length of AB, then go to the construction of a right triangle with AP as the side. The side lengths of AB can be obtained by making $BH \perp PM$ at H. Then the side lengths of AH and HB can be found separately. Further, get the area of the square.

Solution: As shown in the figure 13, rotate $\triangle ABP$ around point B clockwise by 90° to get $\triangle CBM$, connect PM, and pass point B make $BH \perp PM$ in H.

$$\begin{aligned}
 &\because PC = 4 \text{ and } PA = CM = 2\sqrt{3}, \\
 &\therefore PC^2 = CM^2 + PM^2, \\
 &\because \angle BPM = \angle BMP = 45^\circ, \\
 &\therefore \angle APB + \angle BPM = 180^\circ, \therefore A, P, M \text{ are co-linear}, \\
 &\because BH \perp PM, \\
 &\therefore AH = 2\sqrt{3} + 1, \\
 &\therefore AB^2 = AH^2 + BH^2 = (2\sqrt{3} + 1)^2 + 12 = 14 + 4\sqrt{3}, \\
 &\therefore \text{The area of square ABCD is } 14 + 4\sqrt{3},
 \end{aligned}$$

Example 2: (2020 – Xianning) Definition: A quadrilateral with a set of opposite angles congruent to each other is called a pair of congruent quadrilaterals.

(1) If quadrilateral ABCD is a congruent quadrilateral, then the sum of the degrees of $\angle A$ and $\angle C$ is 90° or 270° .

(2) Figure 14 shows that MN is the diameter of $\odot O$, points A, B, and C are on $\odot O$, and AM and CN intersect at point D. Quadrilateral ABCD is a pair of residual quadrilaterals.;

(3) As shown in Figure 15, in the pair of residual quadrilateral ABCD, $AB = BC$ and $\angle ABC = 60^\circ$, what is the quantitative relationship between the lines AD, CD and BD? Write a conjecture and give a reason.

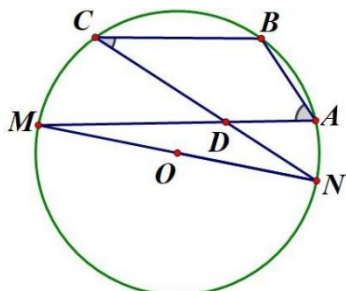


Figure 14

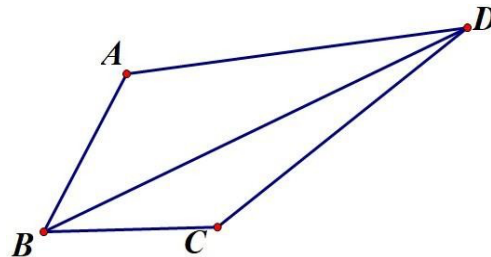


Figure 15

(1) Solution:

$\because$ quadrilateral ABCD is a pair of residual quadrilaterals,

$\therefore \angle A + \angle C = 90^\circ$ or $\angle A + \angle C = 360^\circ - 90^\circ = 270^\circ$,

(2) Proof:

$\because$ MN is the diameter of $\odot O$, points A, B, and C are on $\odot O$,

$\therefore \angle BAM + \angle BCN = 90^\circ$,

So $\angle BAD + \angle BCD = 90^\circ$,

$\therefore$ quadrilateral ABCD is a pair of residual quadrilaterals.

(3)

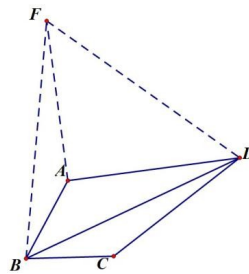


Figure 16

Methodological analysis: This question examines the co-vertex rotation model, in which AD, CD, and BD have a common point D, and $\angle ABC$ is a special angle. With the help of the way and method of the previous problem, it is beneficial if the three-line segments can form a triangle or establish a relationship between two of them. First, establish the relationship between two of the line segments so that the line segments remain unchanged and consider using the rotation transformation. From $AB = BC$, $\angle ABC = 60^\circ$, $\triangle BCD$ rotating around point B and transforming BD to BF, we get equilateral triangle BFD, which in turn gives us $\angle FAD = 90^\circ$ and $AF = CD$, combined with the Pythagorean theorem in the right triangle FAD can get the equivalence of AD, CD, BD, and then carry out the equivalent transformation can be. As shown in Figure 16.

Solution:

$\because AB = BC$.

$\therefore$ rotate $\triangle BCD$ around point B counterclockwise by 60° to get $\triangle BAF$, connect FD

$\therefore \triangle BCD \cong \triangle BAF$, $\angle FBD = 60^\circ$

$\therefore \triangle BFD$ is an equilateral triangle,

$\angle FBD + \angle BFA + \angle ADB + \angle AFD + \angle ADF = 180^\circ$,

$\therefore \angle AFD + \angle ADF = 90^\circ$, $\therefore \angle FAD = 90^\circ$.

$\therefore AD^2 + AF^2 = DF^2$, so $AD^2 + CD^2 = BD^2$.

The problem constantly goes through the positive and negative theorems of the Pythagorean theorem to calculate and explore the quantitative relationship between line segments and line segments as well as the search for special angles, which is a constant switch and use of line length and angle comparison to determine which triangle it is. In the questions, students develop the ability to perform operations according to the laws and algorithms; then collect angle data, make judgments, appreciate the information contained in the data, and determine what kind of classification the newly constructed figure is typical co-vertex of the rotation of the topic, understand the quantitative relationship and the law of change of such questions, for the triangle or quadrilateral when the side lengths are equal, its internal appearance of a point and connected to each vertex, refer to the solution of this question, but also to the same type of questions can be applied to the application of the ability to apply freely.

For students who have learned the knowledge of rotational transformation, then students should consciously use the concept, principle and method of rotation to solve the problems of elementary geometric transformation and expand new ways of thinking. Teachers should find more topics that use rotational transformation, such as line and difference problems, angle equivalence

problems, and common vertex problems, and prompt students to use the method of graphical rotation to solve them, so as to enhance students' creative thinking and innovative ability (Jiang, 2019).

CONCLUSION

The logic of rotational transformation solution

In this paper, different types of problems are cited, such as the problem of sum and difference of lines, the problem of proving the equality of two angles and the problem of co-vertex, all thinking in terms of the solution of rotational transformations, which constitute the congruence of two figures, and extend students' problem solving process and develop their core literacy (Wang et al., 2022). In these problems, starting from the topic problem, the information of the condition is activated, by knowing the condition of the equality of the sides, and then finding that there are special angles or that there are identical corresponding angles that are equal, and finally identifying a target triangle to rotate the triangle by a certain angle with a fixed point. Then it is possible to reason out the practice of auxiliary lines, so that the addition of auxiliary lines can be traced to break through the construction problem and improve the reasoning ability.

Current status of developing students' core literacy in teaching geometry problem solving

In the actual problem solving teaching, the position of students as the main body in the learning process is ignored, and the combination of mathematical knowledge and the cultivation of students' core literacy is not achieved. For the geometric transformation topics are bent on making students do auxiliary lines, while the method of doing auxiliary lines is to do more problems to find the feeling. This kind of misunderstanding and wrong teaching method in mathematics teaching plays a serious hindrance to enhance students' core literacy. Therefore, developing core literacy in mathematics is a facilitator for solving mathematical problems.

How teachers can penetrate core mathematical literacy in teaching rotational transformation problems

1. The questions of elementary geometric transformations usually require proof of the quantitative relationships and the solutions of the corresponding sides or angles. Teachers should combine the standards of the compulsory mathematics curriculum with a deep grasp of the concept of parenting to guide students in understanding or expressing quantitative relationships and to promote the establishment of number sense (Tran et al., 2016).

2. The development of spatial concepts in elementary geometric transformations focuses on abstracting the figure, imagining orientation and position relationships, describing the movement and change of the figure and then constructing new auxiliary lines through rotational transformations to facilitate the completion of the topic. Teachers should explain clearly to students the practice of auxiliary lines so that they can understand the meaning of making them.
3. The ability to explore ideas through given conditions in a plane geometry problem is very important, and teachers should let students do more analysis and conjecture. Ask students about the reasons for their ideas in class, and develop the ability to do so. Mathematics is a subject that requires very rigorous logic, and teachers should guide students to justify their conclusions and analyze them appropriately in order to develop their reasoning skills.
4. Students will be guided to categorize the topics of rotation type, discover the law of change, build and solve the model, get the solution of various types of problems of rotation model in different categories, find the result, and discuss the result, and complete the meaning of multiple solutions and multiple solutions to one (Chen, 2016). Therefore, teachers should guide students to pay attention to the cross-relationship of knowledge and the penetration of mathematical thinking in order to improve the efficiency of students' mathematics review for the HKCEE, to solve mathematical problems through a variety of methods, and to achieve multiple solutions to one problem and multiple solutions to one (wang, 2009).
5. Awareness of application. For the knowledge of rotational transformations learned, consciously use the concepts, principles and methods of rotation to solve problems of elementary geometric transformations and expand new ways of thinking. Teachers should find more topics that use rotational transformations and prompt the use of the rotation of figures to solve them.
6. Awareness of innovation. Students discovering and asking their own questions is the basis of innovation; independent thinking is the core of innovation. For a class of problems, students should disseminate their thinking to constantly seek different solutions, and eventually categorize the problems so that they can learn a problem to understand a class of problems. The sense of innovation is not only in doing problems reflecting different solutions, as a teacher should also be infiltrated in the lecture, and constantly disseminate thinking. Change the form of lecture, such as the use of geometry drawing board, to show students the process of graphical movement, so that students intuitively feel, solve the problem more clearly.

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